

# Simulation of electron energy loss spectra of nanomaterials with linear-scaling density functional theory

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Atomic Scale Materials Microscopy: Theory Meets Experiment  
York 27/6/17

## Functional

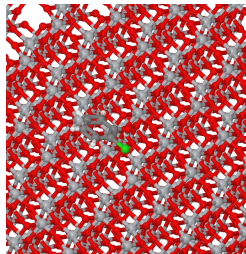
- Photocatalysis
- Photovoltaics

## Economic

- Abundant
- Cheap
- Non-toxic

## Defects

- What surface structures are present?
- Have we made what we intended to make?



## How Can Theory Help?

- Identify likely stable structures (AIRSS etc.)
- Predict experimentally measurable properties

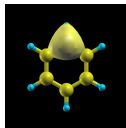
# Linear Scaling DFT - ONETEP

LS-DFT  
for EELS

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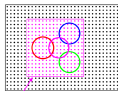
## NGWF

Non-orthogonal Generalised Wannier Function - spatially localised basis function, itself expressed in terms of periodic sinc (PSinc) functions



## FFT Box

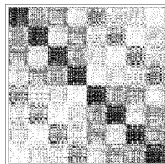
Segment of the cell containing an NGWF and all NGWFs which overlap it



FFT box  
© J. G. Ángels, D. A. M. Stewart, J. E. Pople, C. J. Cramer & M. J. Frisch, Chem. Phys. Lett. 288, 143 (1998)  
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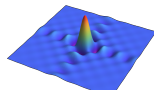
## Density Kernel

Used to express the density matrix in terms of NGWFs  $\rho(r, r') = \phi_\alpha(r) K^{\alpha\beta} \phi_\beta(r')$



## Cut off

The PSinc plane wave basis is truncated to waves with an energy less than the cut off energy



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LS-DFT for EELS

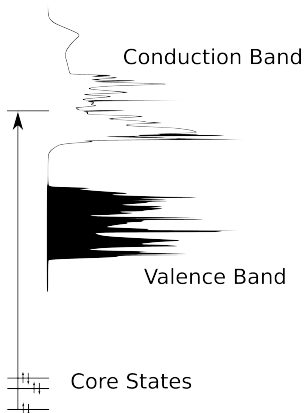
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# Modeling EELS

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- Fermi's Golden Rule
- Dipole approximation
- $\epsilon_2(\omega) = \frac{1}{\Omega} \sum_c \sum_i |\langle \psi_i | \mathbf{r} | \psi_c \rangle|^2 \delta(E_i - E_c - \omega)$
- $c$  runs over core states,  $i$  over conduction band states
- Compute matrix elements like  $|\langle \psi_i | \mathbf{r} | \psi_c \rangle|$
- Focus on single core state
- Use OPTADoS to convert matrix elements into spectrum
- Various broadening schemes available



# Conduction Band

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- In ground state calculation NGWFs and Kernel optimised to represent valence manifold
- Valence NGWFs bad at representing the conduction manifold
- A separate kernel and NGWF set must be optimised
- $\rho_c(r, r') = \chi_\alpha(r) K_c^{\alpha\beta} \chi_\beta(r')$
- Optimisation non self consistent  $\rightarrow$  faster

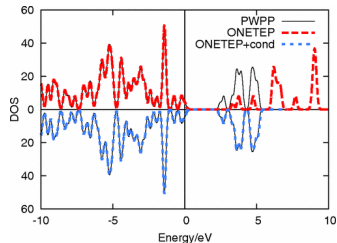


Figure: Improvement in accuracy of conduction DoS when a separate kernel and NGWF set are used. From<sup>1</sup>

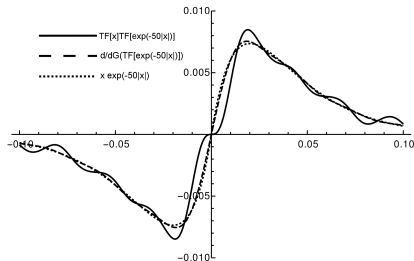
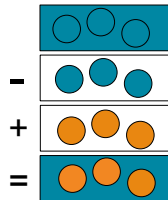
<sup>1</sup>Ratcliff, Laura E., Nicholas DM Hine, and Peter D. Haynes. "Calculating optical absorption spectra for large systems using linear-scaling density functional theory." *Physical Review B* 84, no. 16 (2011): 165131.

# Projector Augmented Wave

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- Need correct wavefunction near the nucleus to do EELS calculations.
- PAW - Projector Augmented Wave, method due to Blöchl used to retrieve all electron properties from a pseudo potential calculation
- $|\psi^{KS}\rangle = |\tilde{\psi}^{KS}\rangle + \sum_i \langle \tilde{p}_i | \tilde{\psi}^{KS} \rangle (|\varphi_i\rangle - |\tilde{\varphi}_i\rangle)$
- Form core-position kets  $\mathbf{r}|\psi_c\rangle$  using Fourier space method

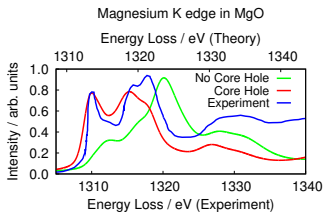


# Core Holes

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- Electron excited to the CB leaves hole
- Electron and hole interact, pronounced in insulators (reduced screening)
- Must account for this
- One solution is to modify PAW dataset used for excited atom
- This is only an approximation
- Can include fractional core holes
- Example configuration strings for carbon:  $1s^2 2s^2 2p^4 \rightarrow 1s^1 2s^2 2p^5$   
 $1s^2 2s^2 2p^4 \rightarrow 1s^{1.5} 2s^2 2p^{4.5}$



- We follow the approach of Mizoguchi<sup>1</sup>
- Compute ground state energy for system with and without core hole
- In the core hole case the excited electron is included
- Energy difference of these two calculations estimates correction to conduction band offset due to core hole
- Use results for dataset generation to include core orbital effects

## Edge Offset Energy

Compute Using:

$$\begin{aligned} E_{\text{edge}} = & (E_{\text{sys+ch+e}} - E_{\text{sys,gs}}) \\ & + (E_{\text{aeatom+ch+e}} - E_{\text{aeatom,gs}}) \\ & - (E_{\text{psatom+ch+e}} - E_{\text{psatom,gs}}) \end{aligned}$$

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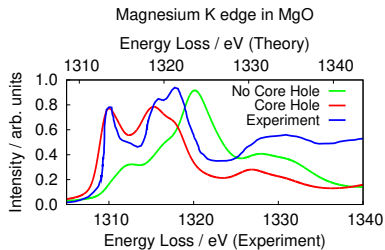
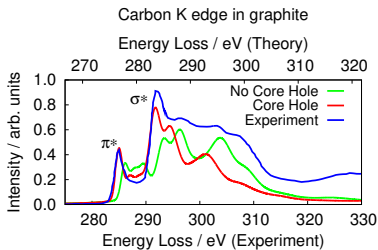
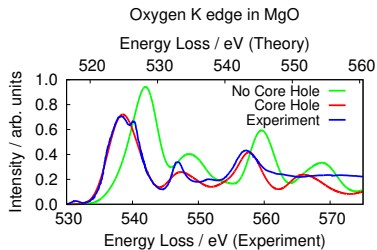
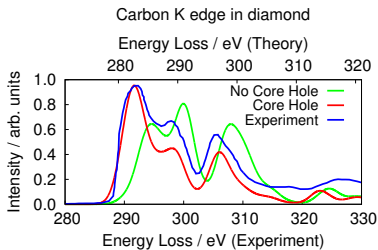
<sup>1</sup>Mizoguchi, Teruyasu, Isao Tanaka, Shang-Peng Gao, and Chris J. Pickard. "First-principles calculation of spectral features, chemical shift and absolute threshold of ELNES and XANES using a plane wave pseudopotential method." *Journal of Physics: Condensed Matter* 21, no. 10 (2009): 104204.



# Some Simple Tests

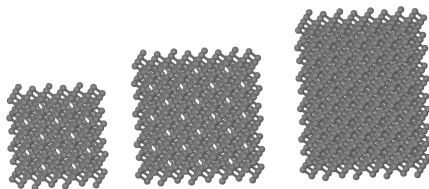
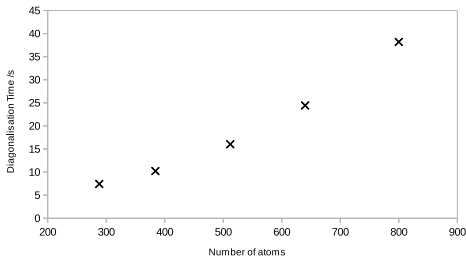
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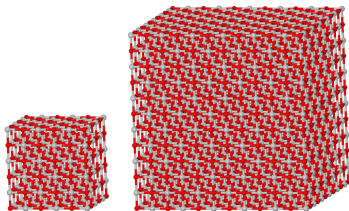
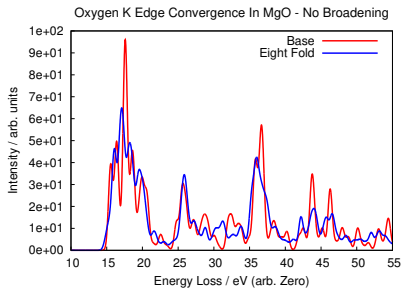
# Effort Scaling

- Ground state calculation:  
 $\mathcal{O}(n)$
- We need Kohn-Sham  
conduction wavefunctions
- Use plain diagonalisation:  
 $\mathcal{O}(n^3)$
- ONETEP's minimal basis  
keeps the matrices  
manageable for systems with  
up to several thousand  
atoms.

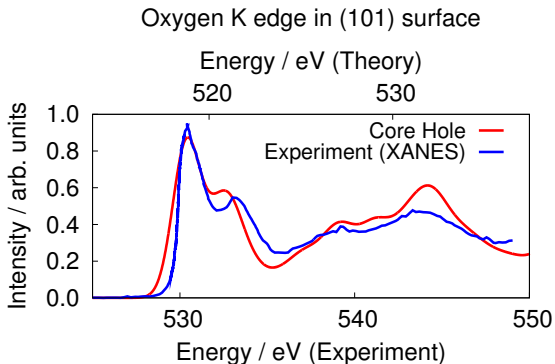


# Size Convergence

- Great advantage of ONETEP - large cells
- We can study the convergence of spectra with respect to simulation cell size
- We find that simulations at the 200 atom scale are well converged



## Comparison to X-ray Absorption Near Edge Spectroscopy<sup>1</sup>

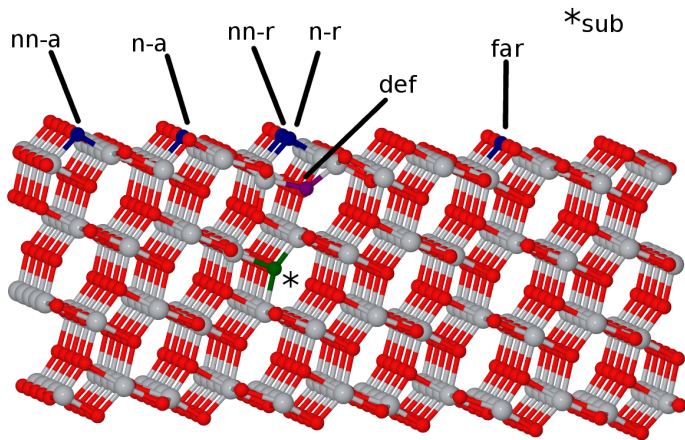


<sup>1</sup>Experimental data from: Thomas, A. G., W. R. Flavell, A. K. Mallick, A. R. Kumarasinghe, D. Tsoutsou, N. Khan, C. Chatwin et al. "Comparison of the electronic structure of anatase and rutile TiO<sub>2</sub> single-crystal surfaces using resonant photoemission and X-ray absorption spectroscopy." *Physical Review B* 75, no. 3 (2007): 035105.

# Adding Defects

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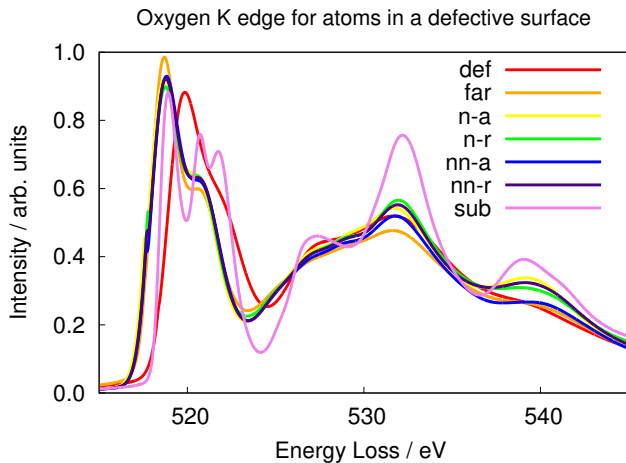
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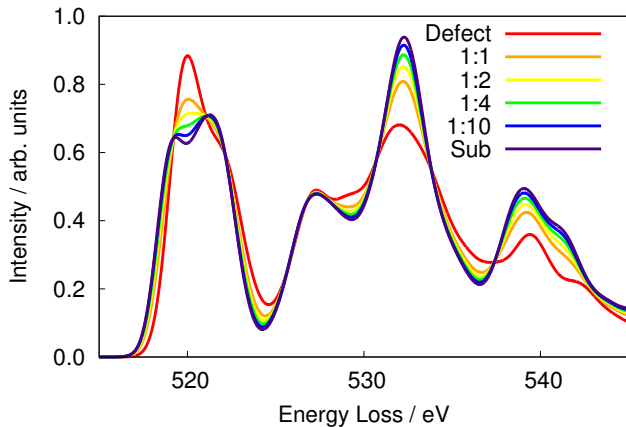
# Initial Results - Very Local?

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### Oxygen K Edge: Mixed Defect-Sub Systems



- Non native defects far easier as their edges will stand out
- EELS really is a very local method
- If we're hunting for defects we wont see anything unless:
  - 1 the beam strikes the size directly.
  - 2 Its helps a lot if the defect leads to split or greatly shifted
- Defects with radically different coordination
- Interstitials likely to be most distinctive intrinsic defect



# Thanks

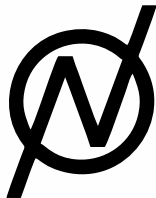
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Laura Ratcliff, who did a great deal of the ground work for this project  
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Cambridge High Performance Computing Service.



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And, of course, all of you for your attention.

