



Nissan Leaf

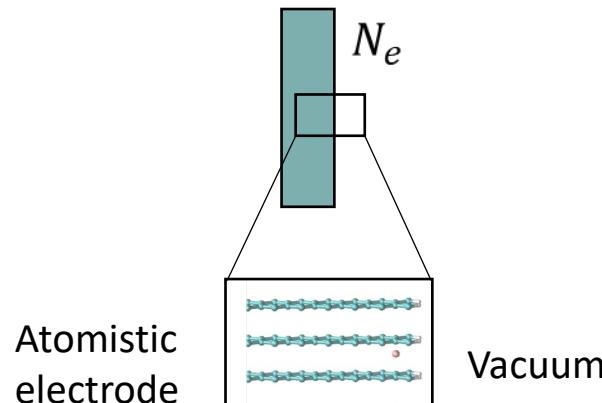
Electrified electrode-electrolyte interfaces from first principles

Arihant Bhandari, Chao Peng, Jacek Dziedzic, Lucian Anton, John R. Owen, Denis Kramer, Chris-Kriton Skylaris

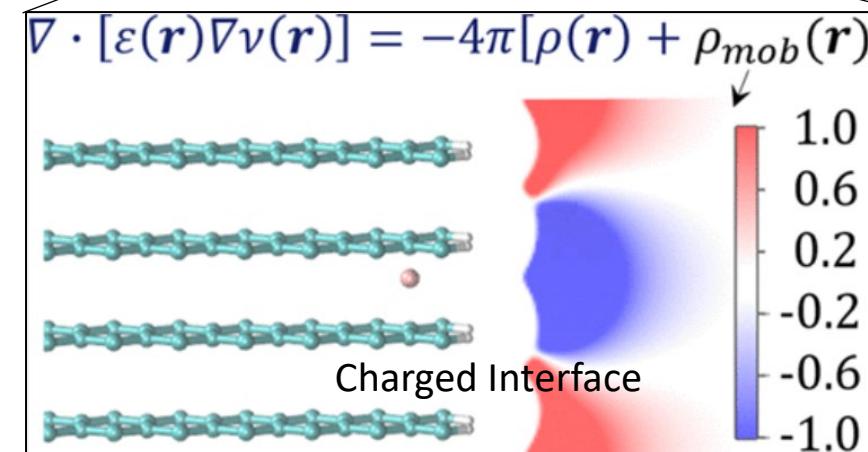
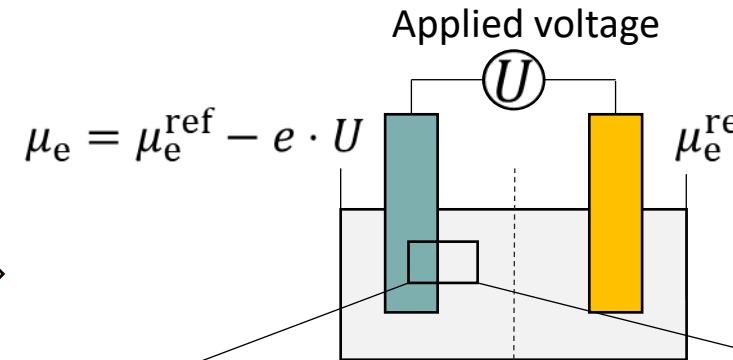
University of Southampton, United Kingdom

Model for simulations of electrified electrode-electrolyte interfaces in ONETEP

Canonical ensemble of electrons
(constant number of electrons)



Grand canonical model for electrochemistry



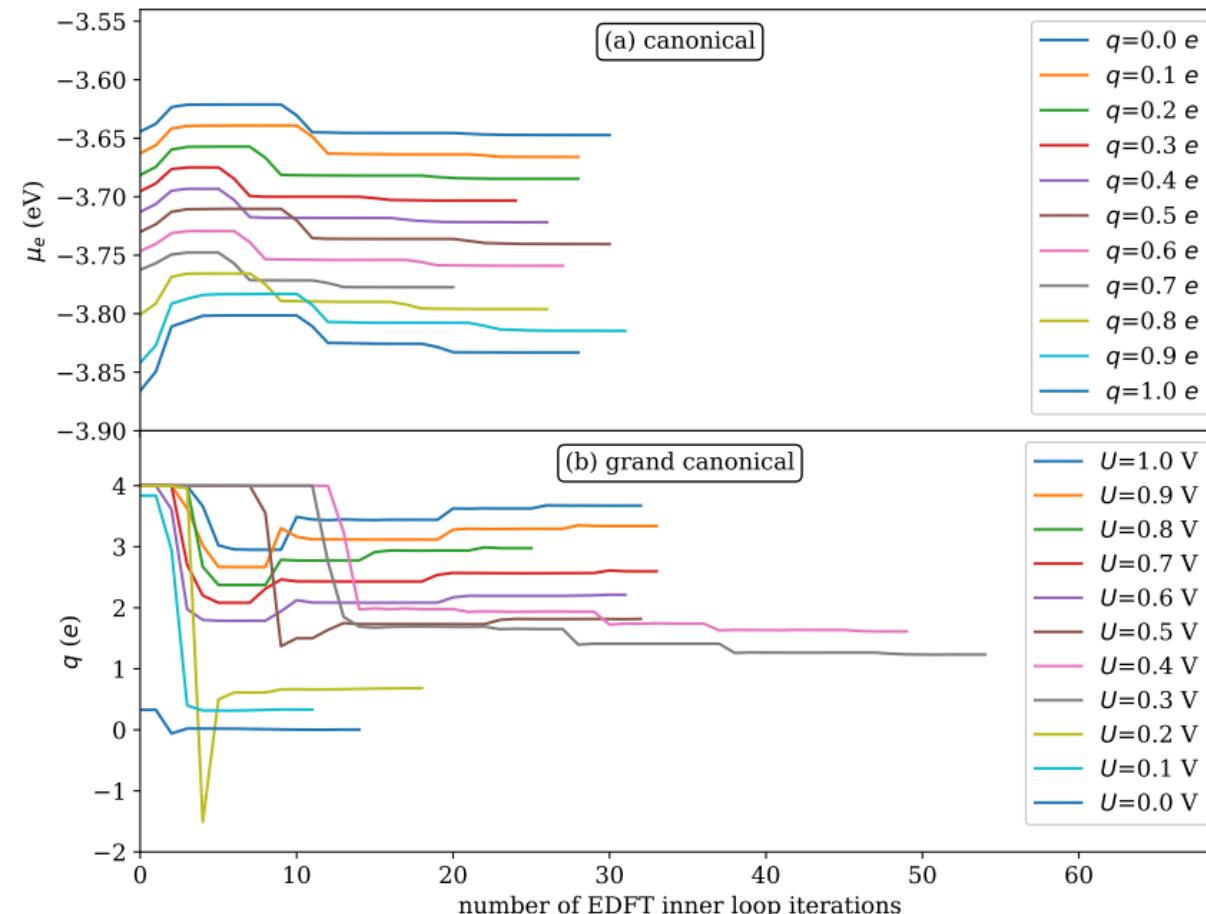
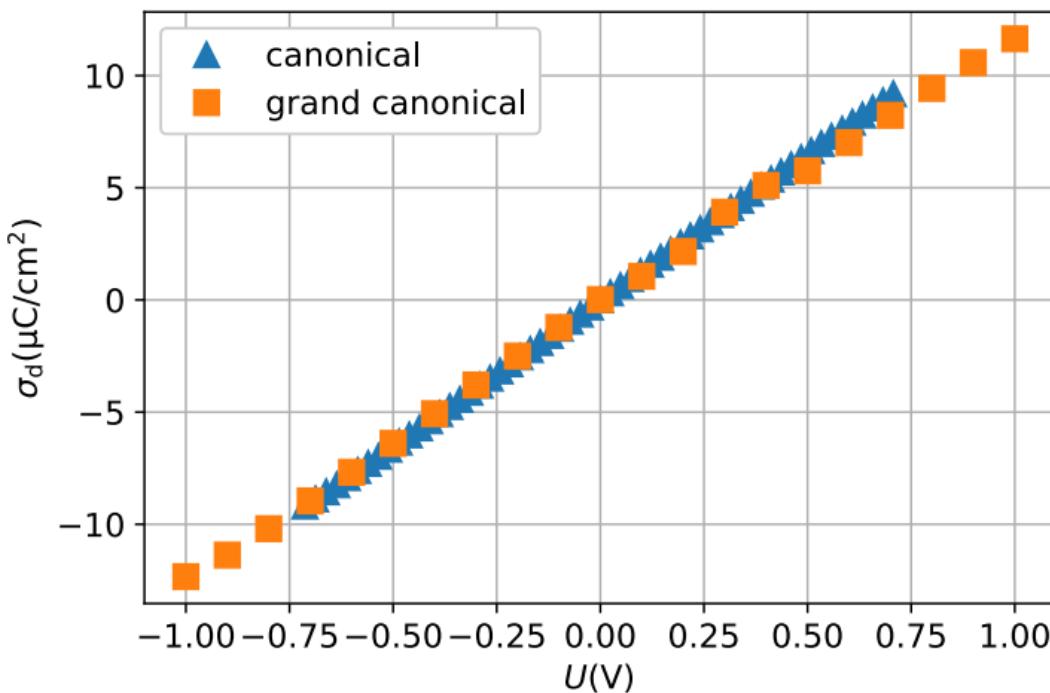
System	μ_e^{ref} , physical scale (eV)
Electron in vac.	0.0
Li^+/Li	-1.39
H^+/H_2 (SHE)	-4.44



Model for simulations of electrified solid-liquid interfaces in ONETEP

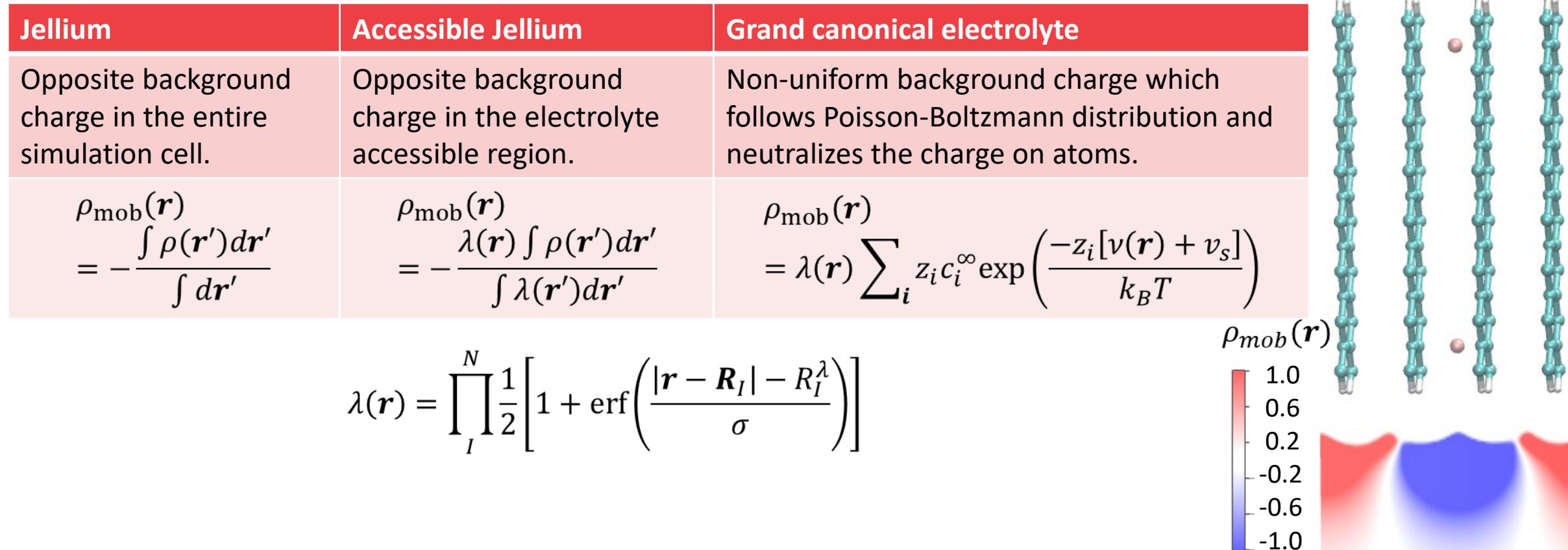
- Ensemble for electrons

Canonical	Grand canonical
Electronically isolated system.	System connected with an electronic reservoir.
Number of electrons and the charge is fixed.	Chemical potential of electrons is fixed.
Chemical potential is found.	Charge is found.



Model for simulations of electrified solid-liquid interfaces in ONETEP

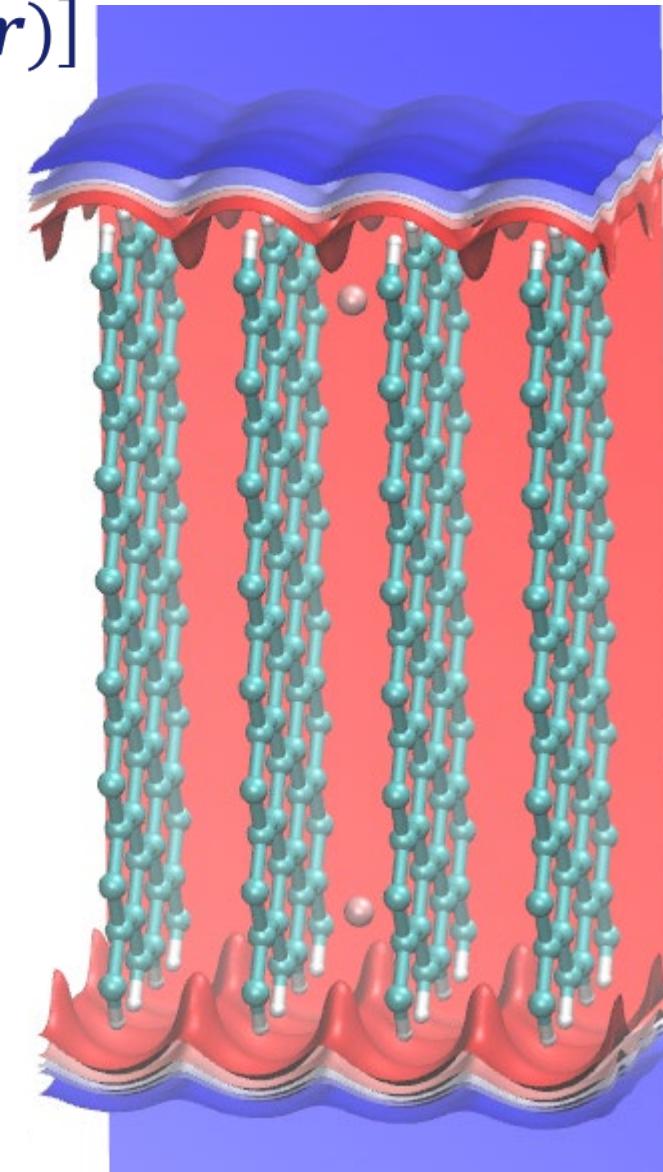
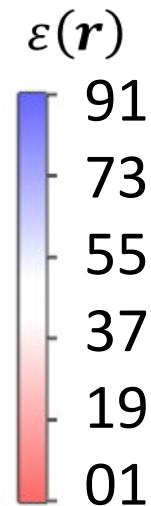
- Electroneutrality $\nabla \cdot [\epsilon(\mathbf{r}) \nabla \nu(\mathbf{r})] = -4\pi[\rho(\mathbf{r}) + \rho_{mob}(\mathbf{r})]$



Model for simulations of electrified solid-liquid interfaces in ONETEP

- Solvent medium $\nabla \cdot [\varepsilon(\mathbf{r}) \nabla v(\mathbf{r})] = -4\pi[\rho(\mathbf{r}) + \rho_{mob}(\mathbf{r})]$

Vacuum	Solvent
Uniform permittivity of 1.0.	Permittivity varies smoothly from 1.0 near the atoms to that of the bulk solvent far away.
No interactions with the atoms.	Cavitation, dispersion and repulsion interaction proportional to the solvent accessible surface area.

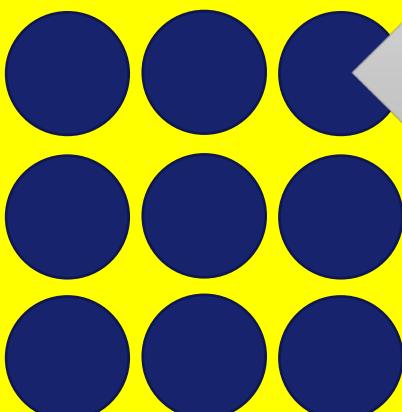


System

Quantum System

$$\rho(\mathbf{r}) = \rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r})$$

$$\rho_e(\mathbf{r}) = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r})$$



Dielectric solvent
and
Continuum electrolyte
 $z_i, c_i(\mathbf{r}), i = 1 \dots p$

$$\Omega = \Omega_e$$

$$+ \Omega_{\text{mf}}$$

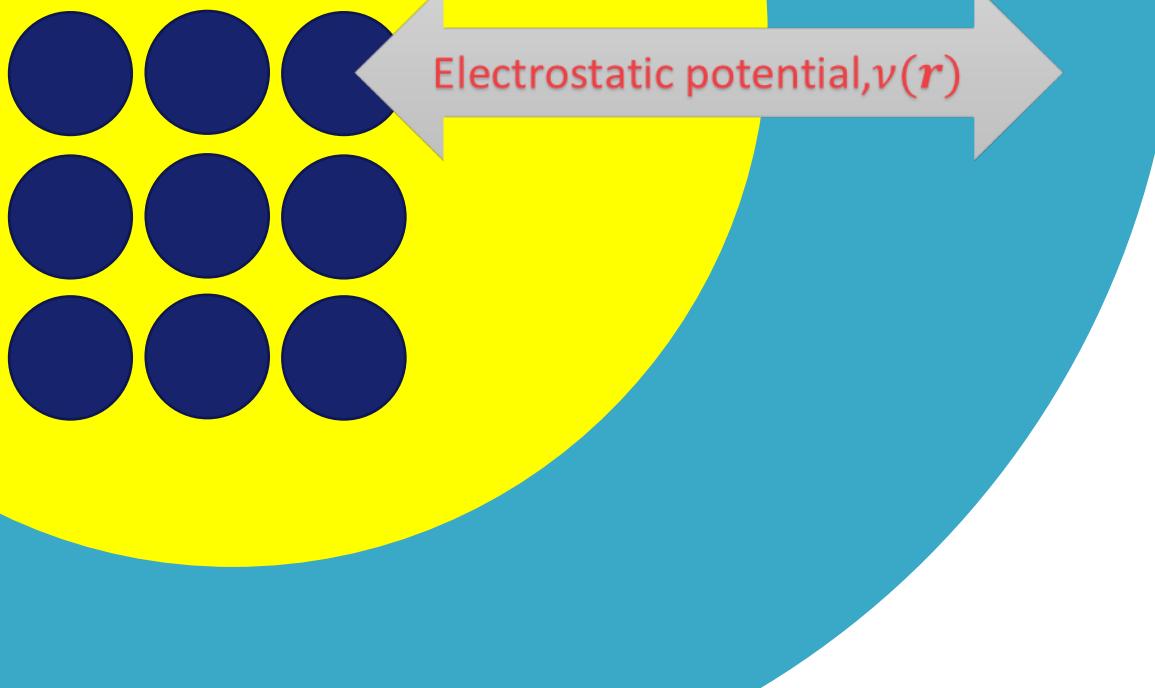
$$+ \Omega_{\text{nmf}}$$

System

Quantum System

$$\rho(\mathbf{r}) = \rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r})$$

$$\rho_e(\mathbf{r}) = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r})$$



Dielectric solvent
and
Continuum electrolyte
 $z_i, c_i(\mathbf{r}), i = 1 \dots p$

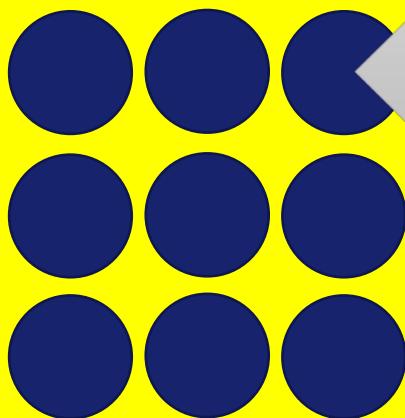
$$\begin{aligned} \Omega = & \sum_k f_k \int \psi_k^*(\mathbf{r}) \left(-\frac{1}{2} \nabla^2 \right) \psi_k(\mathbf{r}) d\mathbf{r} && \text{Kinetic energy} \\ & + k_B T \sum_k f_k \ln f_k + (1 - f_k) \ln f_k && \text{Entropy} \\ & - \mu_e \sum_k f_k && \text{Chemical potential term} \\ & + \sum_k f_k \int \psi_k^*(\mathbf{r}) v_{\text{ps}}(\mathbf{r}) \psi_k(\mathbf{r}) d\mathbf{r} && \text{Pseudopotential contribution} \\ & + E_{\text{xc}} && \text{Exchange Correlation Energy} \end{aligned}$$

System

Quantum System

$$\rho(\mathbf{r}) = \rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r})$$

$$\rho_e(\mathbf{r}) = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r})$$



Dielectric solvent
and
Continuum electrolyte
 $z_i, c_i(\mathbf{r}), i = 1 \dots p$

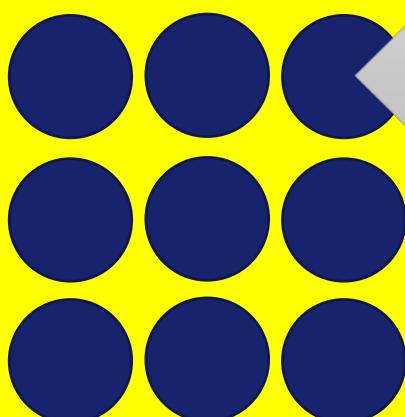
$$\begin{aligned}
 \Omega = & \sum_k f_k \int \psi_k^*(\mathbf{r}) \left(-\frac{1}{2} \nabla^2 \right) \psi_k(\mathbf{r}) d\mathbf{r} && \text{Kinetic energy} \\
 & + k_B T \sum_k f_k \ln f_k + (1 - f_k) \ln f_k && \text{Entropy} \\
 & - \mu_e \sum_k f_k && \text{Chemical potential term} \\
 & + \sum_k f_k \int \psi_k^*(\mathbf{r}) v_{\text{ps}}(\mathbf{r}) \psi_k(\mathbf{r}) d\mathbf{r} && \text{Pseudopotential contribution} \\
 & + E_{\text{xc}} && \text{Exchange Correlation Energy} \\
 & + \frac{1}{2} \int \left[\rho(\mathbf{r}) + \sum_i z_i c_i(\mathbf{r}) \right] v(\mathbf{r}) d\mathbf{r} && \text{Electrostatic energy} \\
 & - k_B T \sum_{i=1}^p \int c_i(\mathbf{r}) d\mathbf{r} && \text{Electrolyte osmotic pressure} \\
 & - k_B T \sum_{i=1}^p \int c_i(\mathbf{r}) \ln \lambda(\mathbf{r}) d\mathbf{r} && \text{Electrolyte accessibility term} \\
 & + k_B T \sum_{i=1}^p \int c_i(\mathbf{r}) \ln \left(\frac{c_i(\mathbf{r})}{c^{\circ}} \right) d\mathbf{r} && \text{Electrolyte entropy} \\
 & - \sum_{i=1}^p \mu_i \int c_i(\mathbf{r}) d\mathbf{r} && \text{Electrolyte chemical potential}
 \end{aligned}$$

System

Quantum System

$$\rho(\mathbf{r}) = \rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r})$$

$$\rho_e(\mathbf{r}) = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r})$$



Dielectric solvent
and
Continuum electrolyte
 $z_i, c_i(\mathbf{r}), i = 1 \dots p$

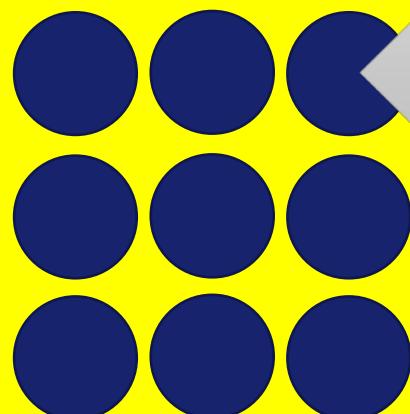
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System

Quantum System

$$\rho(\mathbf{r}) = \rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r})$$

$$\rho_e(\mathbf{r}) = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r})$$



Dielectric solvent
and
Continuum electrolyte
 $z_i, c_i(\mathbf{r}), i = 1 \dots p$

Total free energy,
 $\Omega[\rho_e(\mathbf{r}), c_i(\mathbf{r}), \nu(\mathbf{r})]$

is minimized

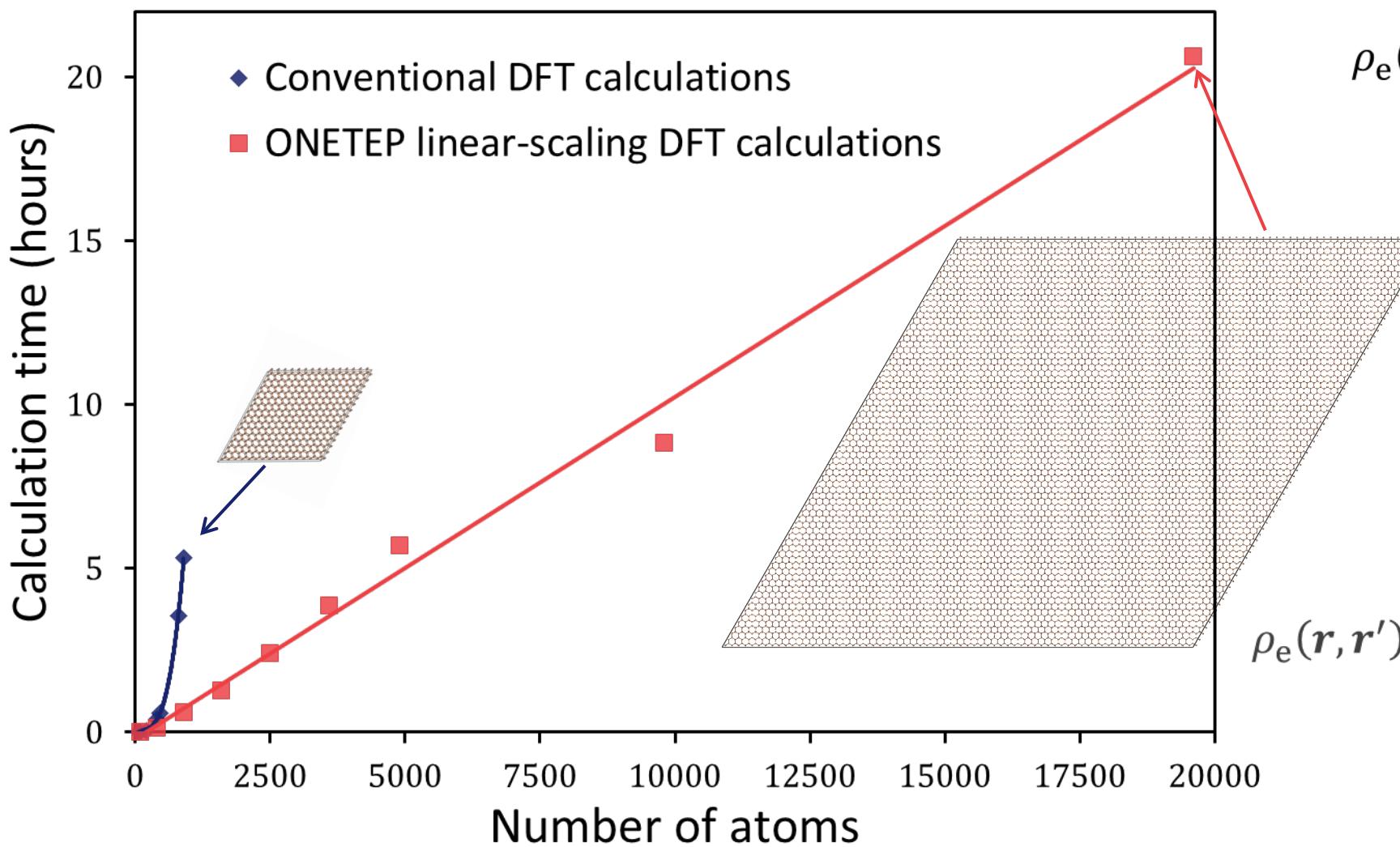
$$\left[-\frac{1}{2} \nabla^2 + \nu_{\text{ps}}(\mathbf{r}) + \nu_{\text{xc}}(\mathbf{r}) + \nu(\mathbf{r}) \right] \psi_k = \varepsilon_k \psi_k$$

$$f_k = \frac{1}{1 + \exp\left(\frac{\varepsilon_k - \mu_e}{k_B T}\right)}$$

$$\nabla \cdot [\varepsilon(\mathbf{r}) \nabla \nu(\mathbf{r})] = -4\pi \left[\rho_e(\mathbf{r}) + \rho_{\text{nuc}}(\mathbf{r}) + \sum_{i=1}^p z_i c_i(\mathbf{r}) \right]$$

$$c_i(\mathbf{r}) = c^0 \lambda(\mathbf{r}) \exp\left(-\frac{z_i \nu(\mathbf{r})}{k_B T} + \frac{\mu_i}{k_B T}\right)$$

ONETEP: quantum atomistic (Density Functional Theory - DFT) program with unique linear-scaling computational effort with the number of atoms



$$\rho_e(\mathbf{r}, \mathbf{r}') = \sum_k f_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r}')$$

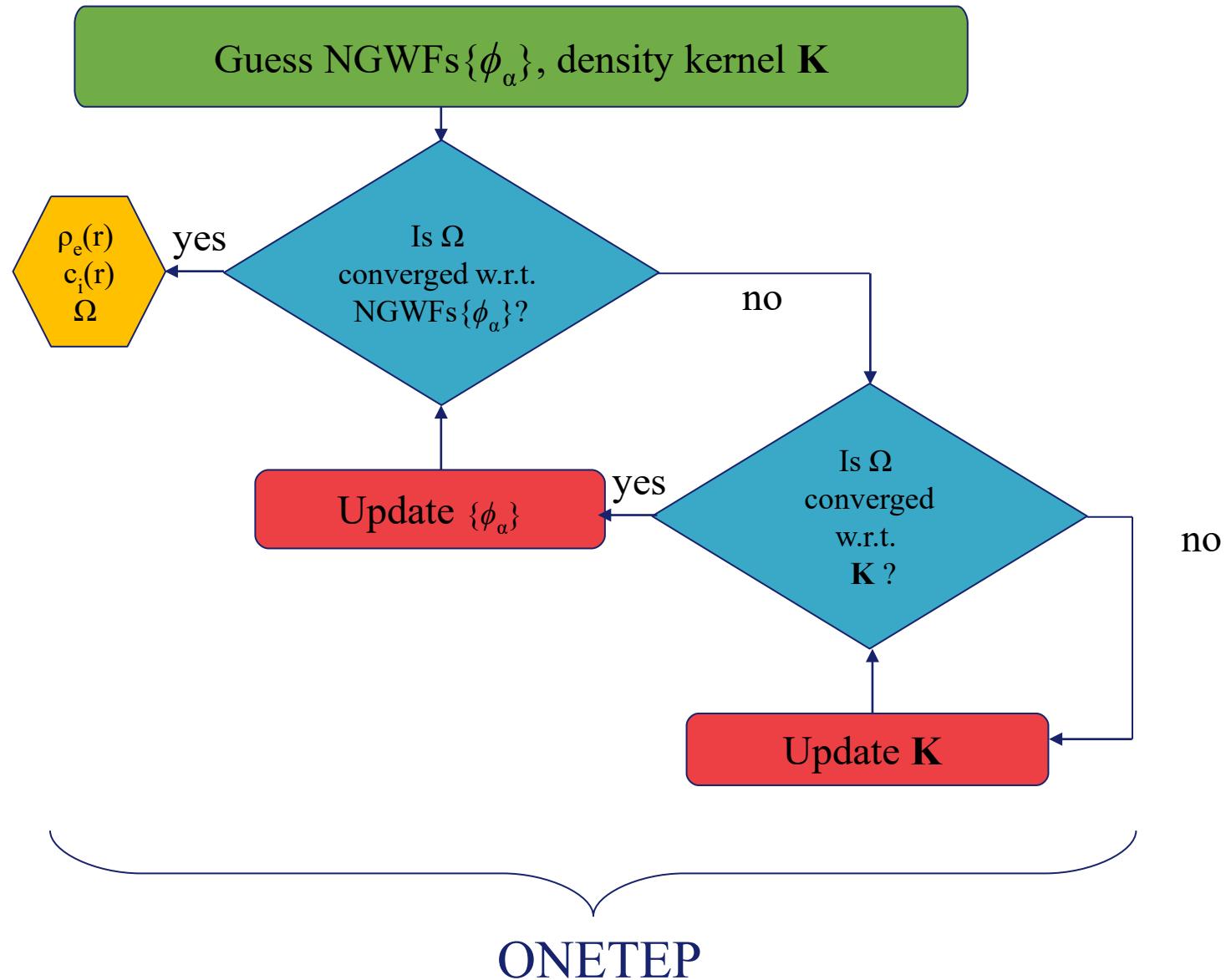
Molecular orbitals

$$= \sum_{\alpha, \beta} \phi_\alpha(\mathbf{r}) K^{\alpha\beta} \phi_\beta(\mathbf{r}')$$

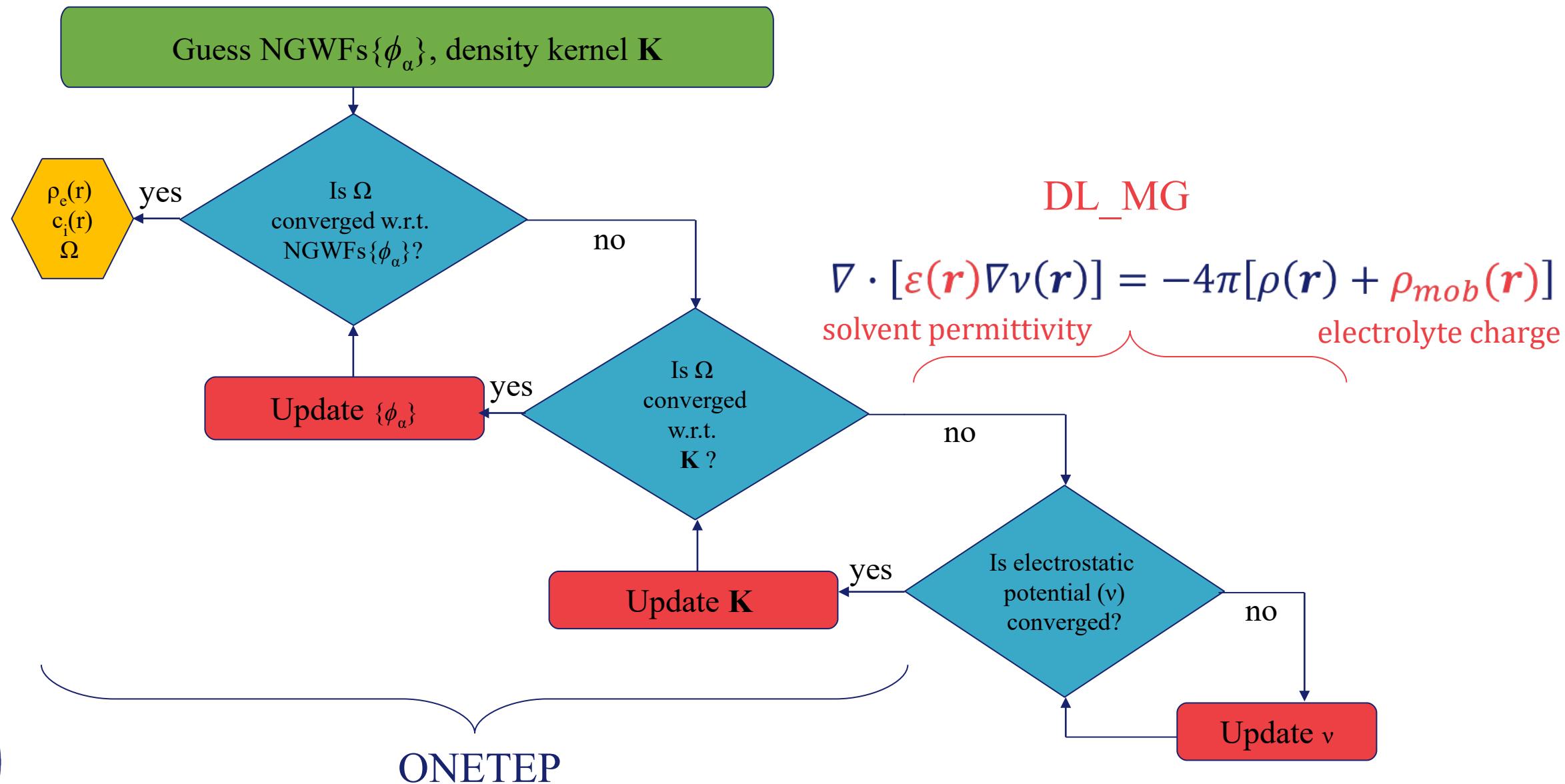
NGWFs

$$\rho_e(\mathbf{r}, \mathbf{r}') \sim e^{-\gamma |\mathbf{r} - \mathbf{r}'|} \rightarrow 0, \text{ as } |\mathbf{r} - \mathbf{r}'| \rightarrow \infty$$

ONETEP Calculation procedure in vacuum

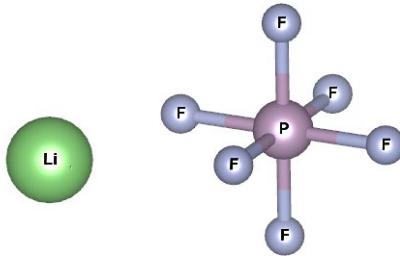


ONETEP Calculation procedure with new electrolyte model

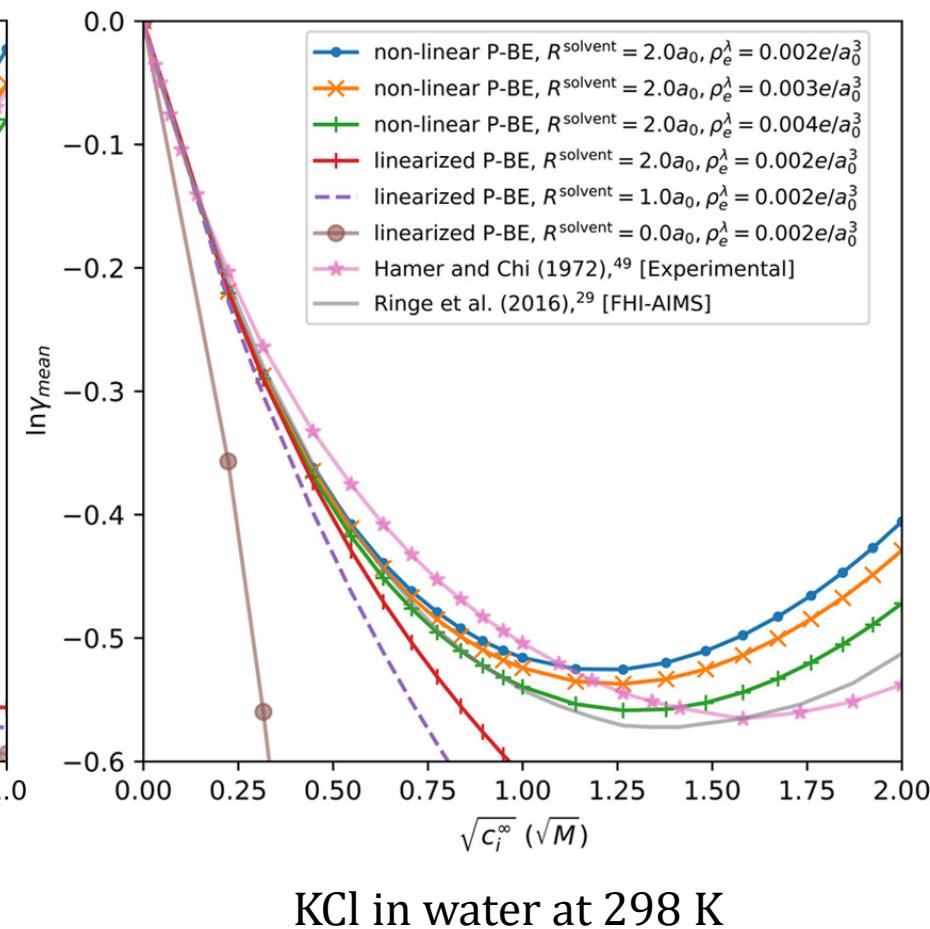
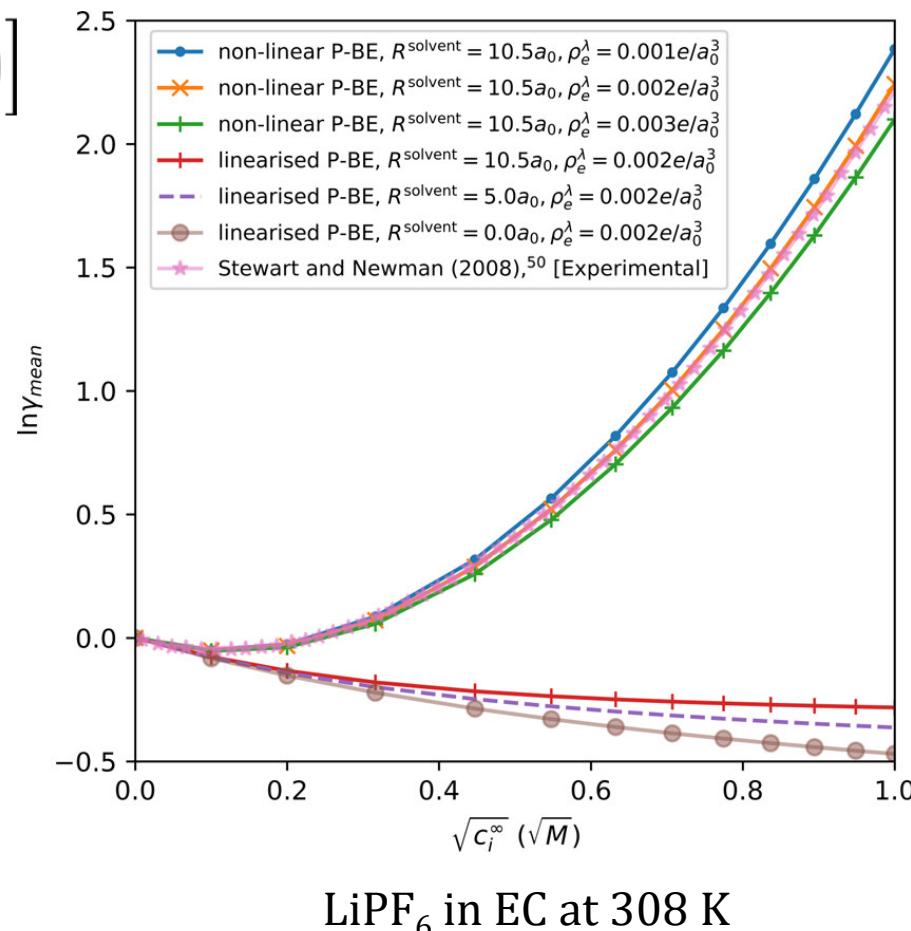


Electrolyte Parameters

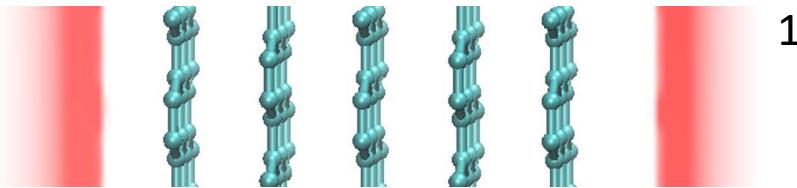
- $\lambda(\mathbf{r}) = \prod_I^N \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{|\mathbf{r}-\mathbf{R}_I| - R_I^\lambda}{\sigma}\right) \right]$
- $R_I^\lambda = R_{\text{solute}}(\rho_e^\lambda) + R_{\text{solvent}}$
- Mean activity coefficients from
- Solvation free energies ($\Delta\Omega_i$)



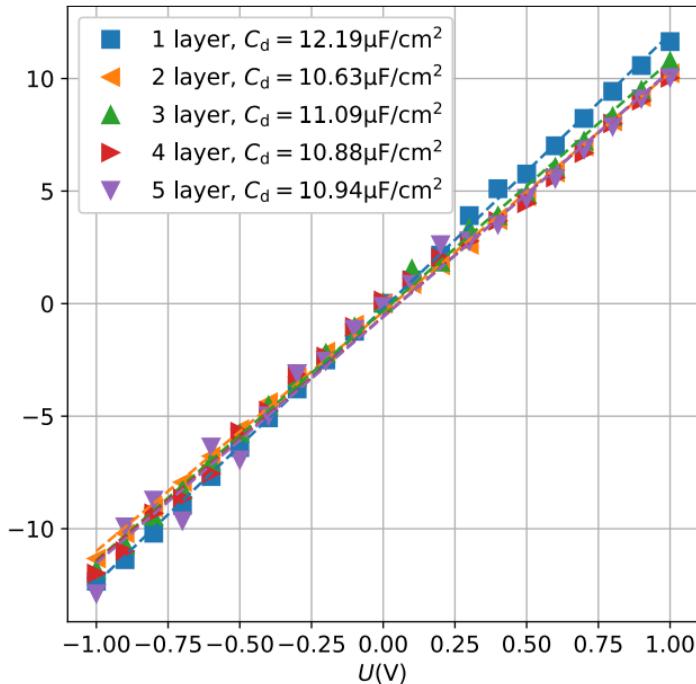
- $\ln \gamma_i = \frac{\Delta\Omega_i(c_i^\infty) - \Delta\Omega_i(c_i^\infty=0)}{k_B T}$
- $\ln \gamma_{\text{mean}} = \frac{1}{p} \sum_{i=1}^p \ln \gamma_i$



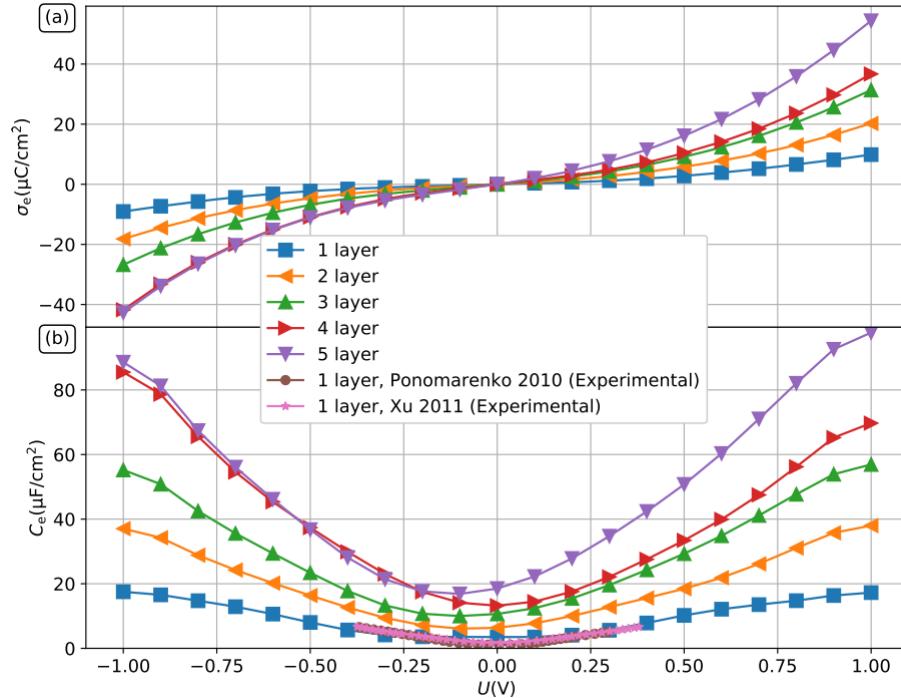
Differential capacitance of few-layer graphene



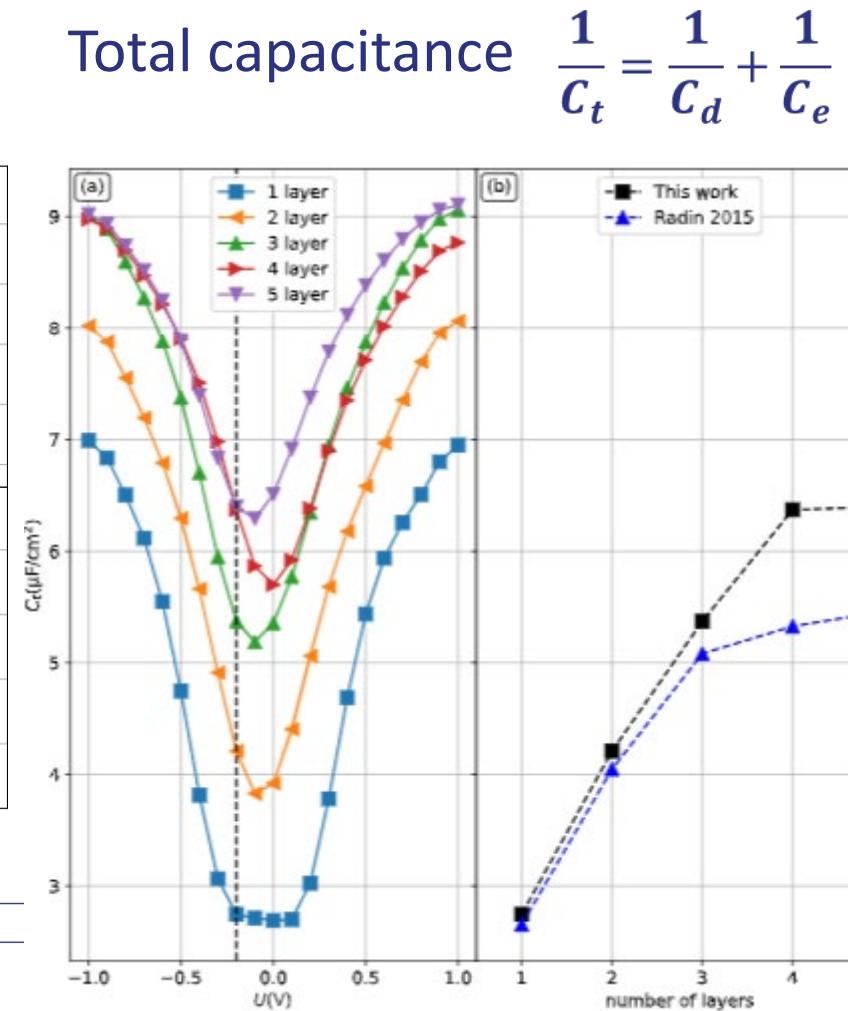
Diffuse layer capacitance



Electronic capacitance



Total capacitance



Electrochemistry from first-principles
in the grand canonical ensemble

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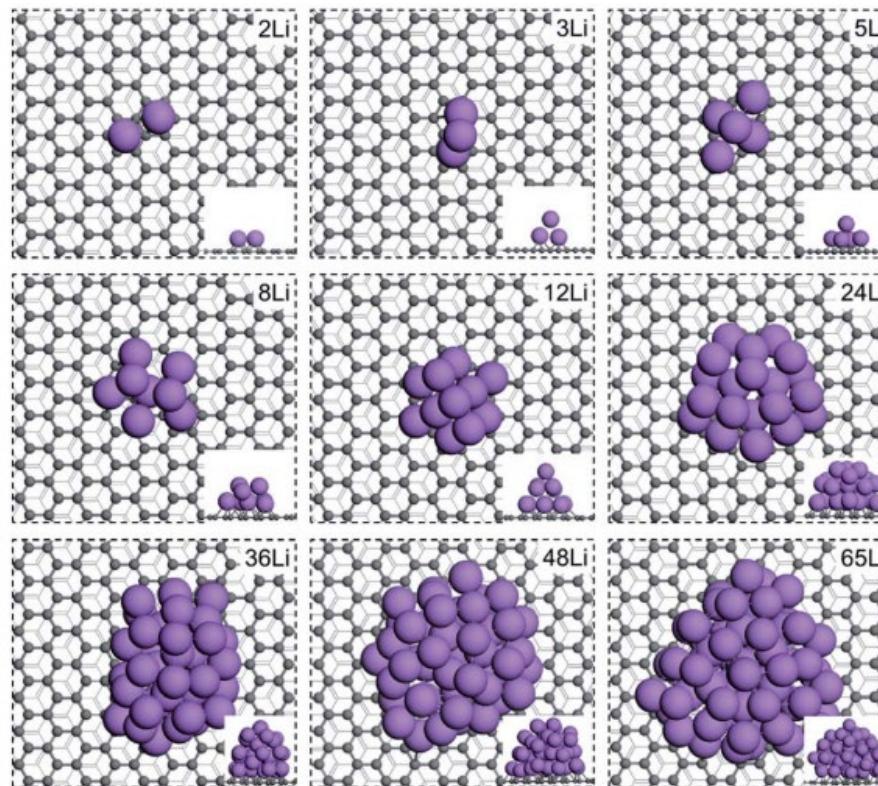
Published Online: 12 July 2021

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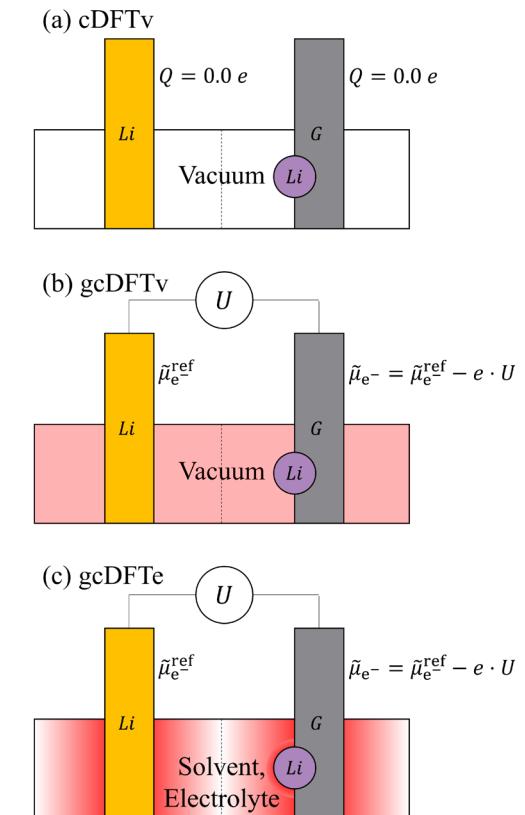
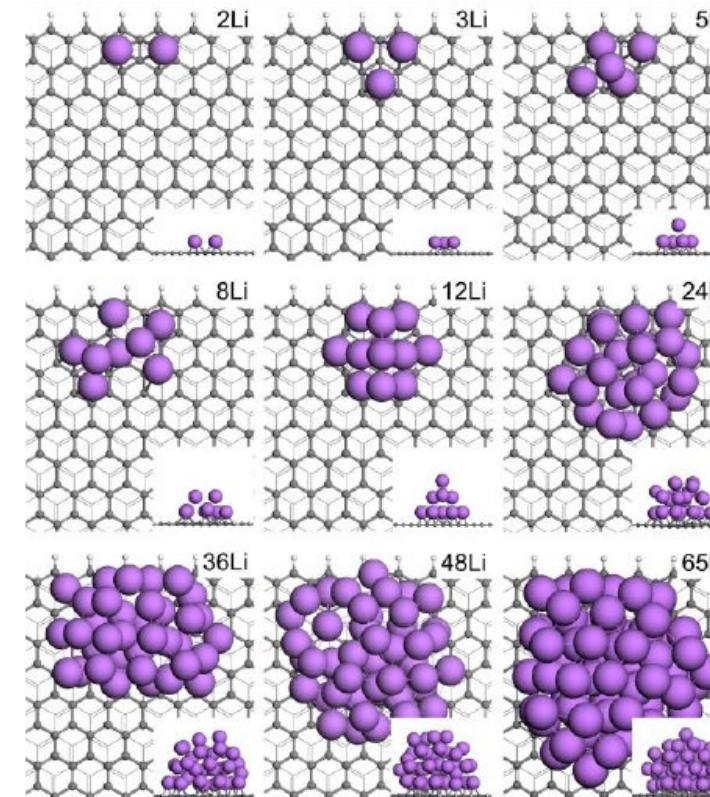


Critical voltage for degradation due to dendrite growth in Li-ion batteries

- On extended basal plane



- Close to zigzag edge



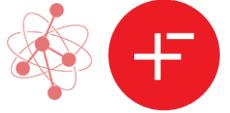
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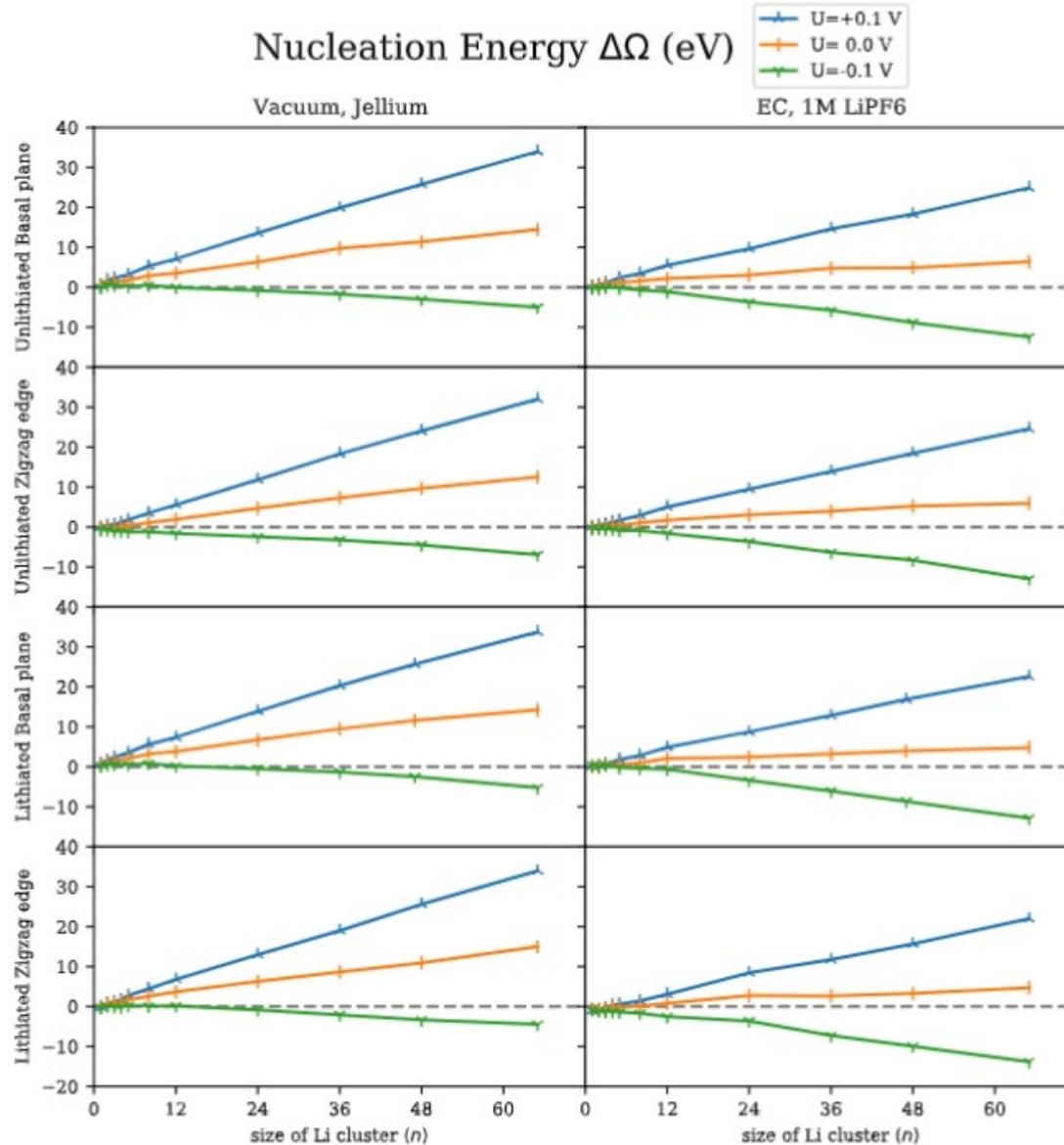
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Nucleation Energy

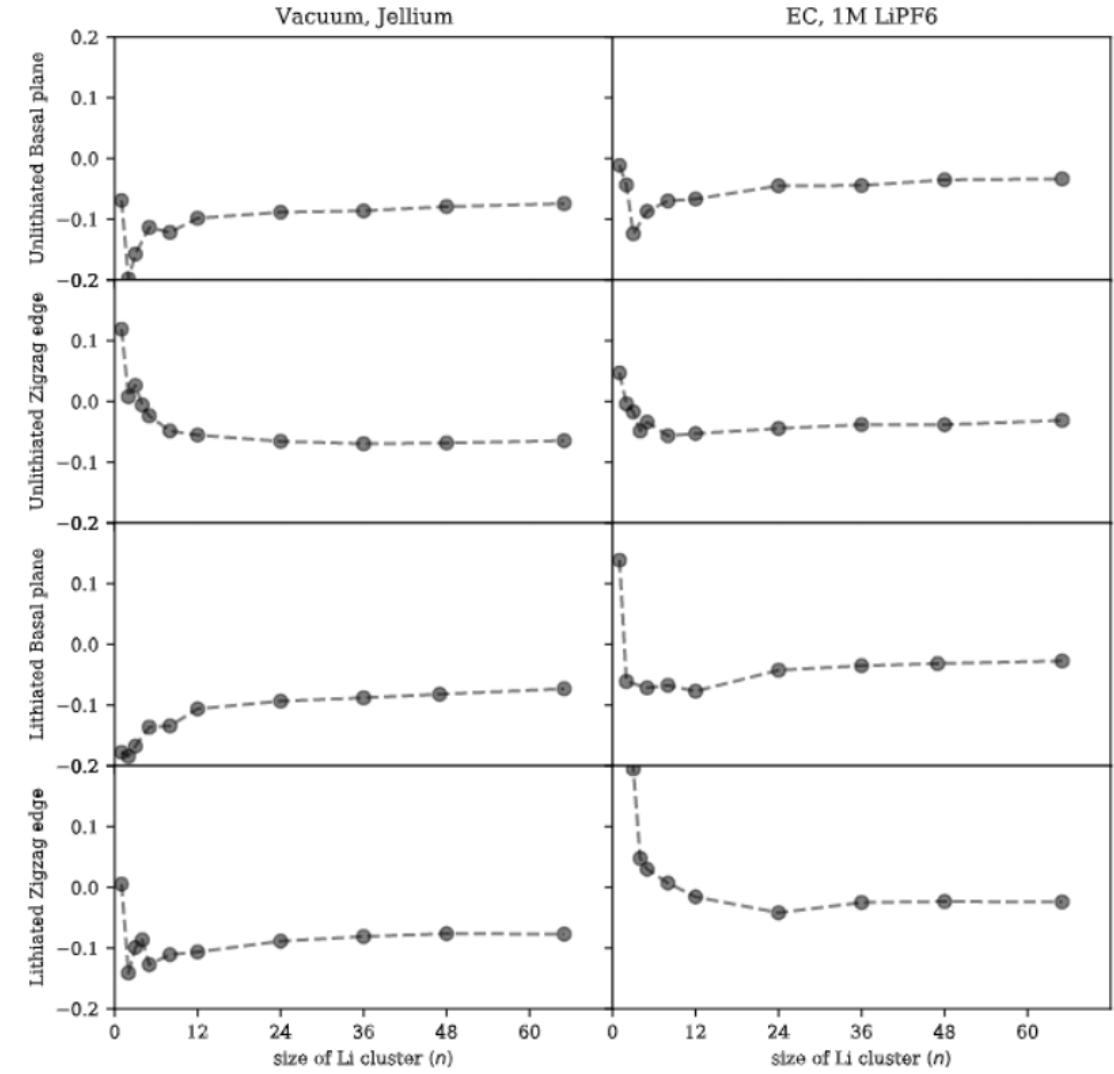


$$n \text{ Li} + G \rightarrow \text{Li}_n G,$$

$$\Delta\Omega(U) = [\Omega_{\text{Li}_n|G}(U) - \Omega_G(U) - n \cdot (\tilde{\mu}_{\text{Li}}^{\text{ref}})]$$



Potential of Zero Nucleation Energy, U_{PZN} (V)



Critical voltage for degradation due to dendrite growth in Li-ion batteries

System	Ensemble	Canonical	Grand canonical		Experiment
	Environment	Vacuum (cDFTv)	Vacuum, Jellium (gcDFTv)	EC, 1M LiPF ₆ (gcDFTe)	
Unlithiated graphite	Basal Plane	-0.65 V	-0.20 V	-0.12 V	-0.15 to -0.04 V Gao et al. Joule 5, 393–414, (2021)
	Zigzag edge	-0.31 V	-0.07 V	-0.06 V	
	Basal Plane	N.A.	-0.18 V	-0.08 V	-0.15 to -0.04 V Gao et al. Joule 5, 393–414, (2021)
	Zigzag edge	-0.47 V	-0.14 V	-0.04 V	

(a) cDFTv: Schematic of a basal plane of graphite with a lithium atom (Li) and a reference point G. The charge at the interface is $Q = 0.0 \text{ e}$. Below is a hexagonal lattice representation of the basal plane.

(b) gcDFTv: Schematic of a basal plane of graphite with a lithium atom (Li) and a reference point G. The charge at the interface is $\tilde{\mu}_e^{\text{ref}}$. A reference potential U is shown. The equation $\tilde{\mu}_{e^-} = \tilde{\mu}_e^{\text{ref}} -$ is indicated. Below is a hexagonal lattice representation of the basal plane.

(c) gcDFTe: Schematic of a basal plane of graphite with a lithium atom (Li) and a reference point G. The charge at the interface is $\tilde{\mu}_e^{\text{ref}}$. A reference potential U is shown. The equation $\tilde{\mu}_{e^-} = \tilde{\mu}_e^{\text{ref}} -$ is indicated. Below is a hexagonal lattice representation of the basal plane.

Scope for improvements

- Accessibility function independent of type of electrolyte ion (i)

$$c_i(\mathbf{r}) = \lambda(\mathbf{r}) c_i^\infty \exp\left[-\frac{z_i \nu(\mathbf{r})}{k_B T}\right]$$

- Point size of electrolyte ions. Finite size effects can be included with a sterically modified Poisson-Boltzmann equation, which limits the maximum concentration (c_{\max}) of electrolyte ions.

$$c_i(\mathbf{r}) = \frac{\lambda(\mathbf{r}) c_i^\infty \exp\left[-\frac{z_i \nu(\mathbf{r})}{k_B T}\right]}{1 - \sum_i \frac{c_i^\infty}{c_{\max}} \left(1 - \exp\left[-\frac{z_i \nu(\mathbf{r})}{k_B T}\right]\right)}$$

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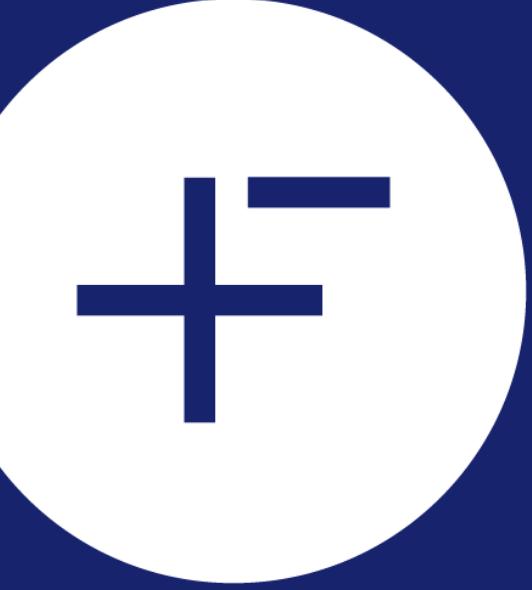
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- Dr. Chao Peng (graphite surface structure models) and Prof. Dr. Denis Kramer
- Prof. Dr. John R. Owen for useful discussions
- Supercomputers: Iridis 5 (Southampton), Archer2, Michael (Faraday Institution) and Young.
- Faraday Institution Multi-Scale Modelling Project (funding)
- Thanks for your attention!



Questions